CENG 511: Algorithms Analysis and Design **09.04.2014**

Problems

1. What is the running time of BFS if we represent its input graph by an adjacency matrix and modify the algorithm to handle this form of input?

2. Give an algorithm that determines whether or not a given undirected graph G = (V, E) contains a cycle. Your algorithm should run in O(V) time, independent of E.

3. Give an example graph which has at least two different topological orderings. Give an example graph which has exactly one topological ordering.

4. There are two types of professional wrestlers: good guys and bad guys. Between any pair of professional wrestlers, there may or may not be a rivalry. Suppose we have n professional wrestlers and we have a list of r pairs of wrestlers for which there are rivalries. Give an O(n + r)-time algorithm that determines whether it is possible to designate some of the wrestlers as good guys and the remainder as bad guys such that each rivalry is between a good guy and a bad guy. If it is possible to perform such a designation, your algorithm should produce it.

5. Consider a rooted DAG (directed, acyclic graph with a vertex – the root – that has a path to all other vertices). Give a linear time (O(|V| + |E|)) algorithm to find the length of the longest simple path from the root to each of the other vertices. (The length of a path is the number of edges on it.)

6. Figure 1 illustrates BFS (Breadth-First Search) on an undirected graph where s is the source. Show how BFS works on the graph of Figure 2 where a is the source. Show all steps as in Figure 1.

7. Figure 3 illustrates DFS (Depth-First Search) on a directed graph where u is the source. Show how DFS works on the graph of Figure 2 where a is the source. Show all steps as in Figure 3.

8. Figure 4 illustrates Kruskal's Algorithm on an undirected graph. Show how Kruskal's Algorithm works on the graph of Figure 6. Show all steps as in Figure 4.

9. Figure 5 illustrates Prim's Algorithm on an undirected graph. Show how Prim's Algorithm works on the graph of Figure 6. Show all steps as in Figure 5.

10. Figure 7 illustrates Bellman-Ford Algorithm on a directed graph with source s. Show how Bellman-Ford Algorithm works on the graph of Figure 9 with source 1. Show all steps as in Figure 7.

11. Figure 8 illustrates Dijkstra's Algorithm on a directed graph with source s. Show how Dijkstra's Algorithm works on the graph of Figure 10 with source 0. Show all steps as in Figure 8.

12. Figure 11 illustrates Floyd-Warshall Algorithm on a directed graph. Show how Floyd-Warshall Algorithm works on the graph of Figure 9. Show all steps as in Figure 11.

13. Let G = (V, E) be a directed graph in which each vertex $u \in V$ is labeled with a unique integer L(u) from the set {1, 2, …, |V|}. For each vertex $u \in V$, let R(u) be the set of vertices that are reachable from u. Define min(u) to be the vertex in R(u) whose label is minimum. Give an O(V+E)-time algorithm that computes min(u) for all vertices $u \in V$.

14. Given an undirected connected graph G = (V,E). For every 2 sets of vertices V_1 and V_2 such as $V_1 \cup V_2 \subseteq V$ we define: distance(u,v) - the length of the shortest path from u to v in G. distance(V_1, V_2) – the minimum length of shortest path between a vertex v_1 in V_1 and v_2 in V_2. (Note: If $V_1 \cap V_2 \neq \emptyset$ then distance(V_1, V_2)= 0). Find distance(V_1, V_2) in O(|V| + |E|) time.

Figures

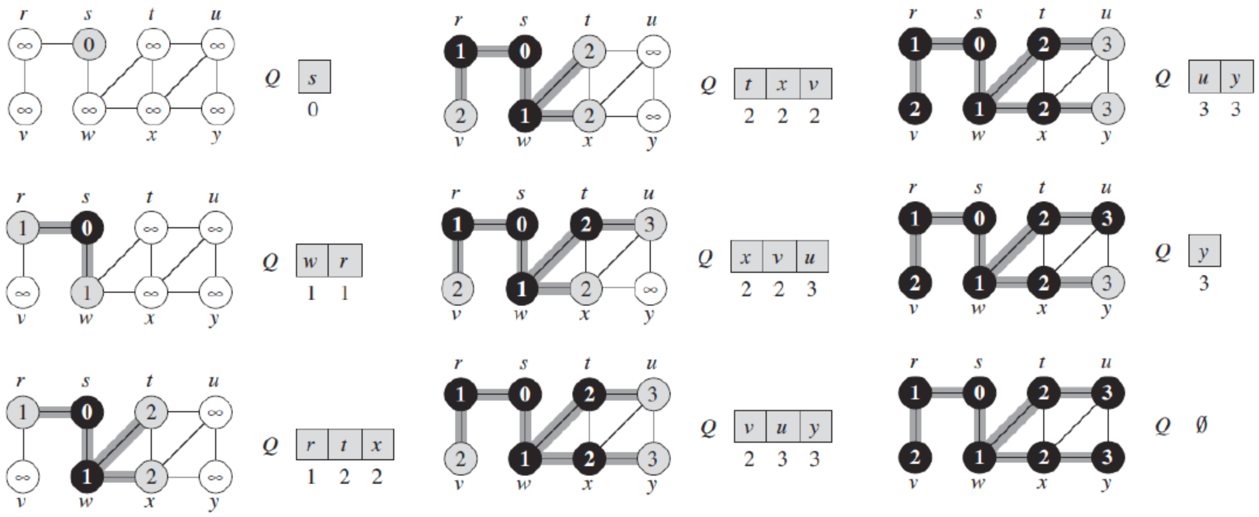


Figure 1

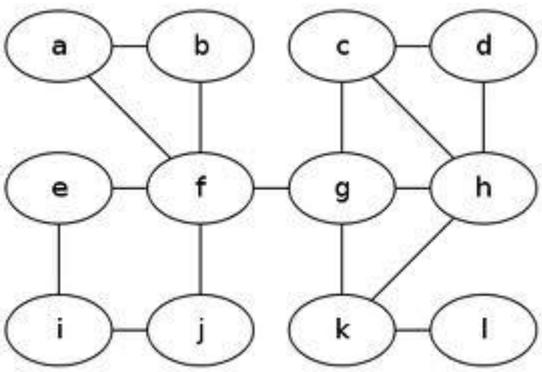
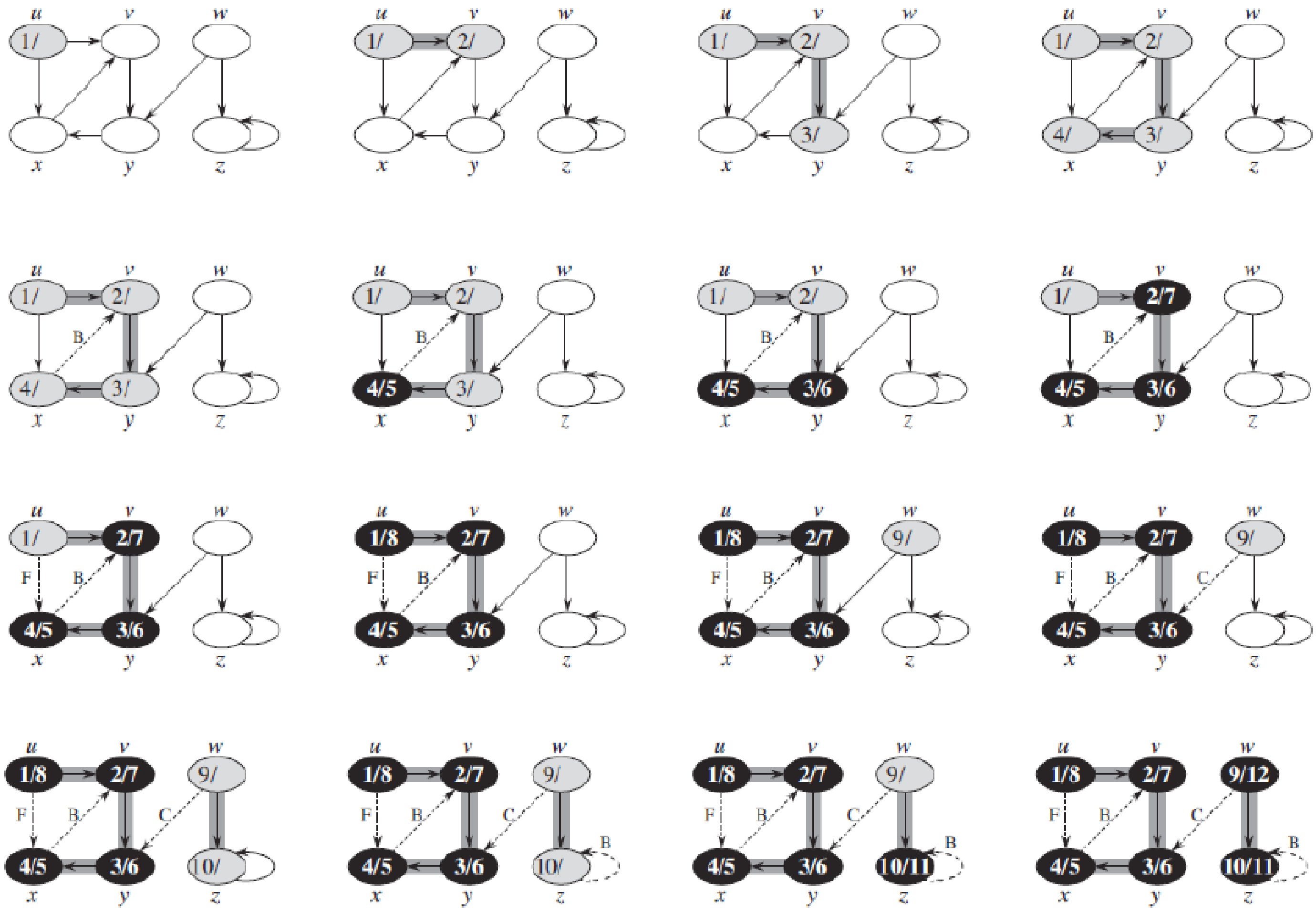


Figure 2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u v w 1/8 2/7 9/ F B C 4/5 3/6 x y z
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u v w 1/8 2/7 9/ F B C B 4/5 3/6 10/ x y z
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Figure 3

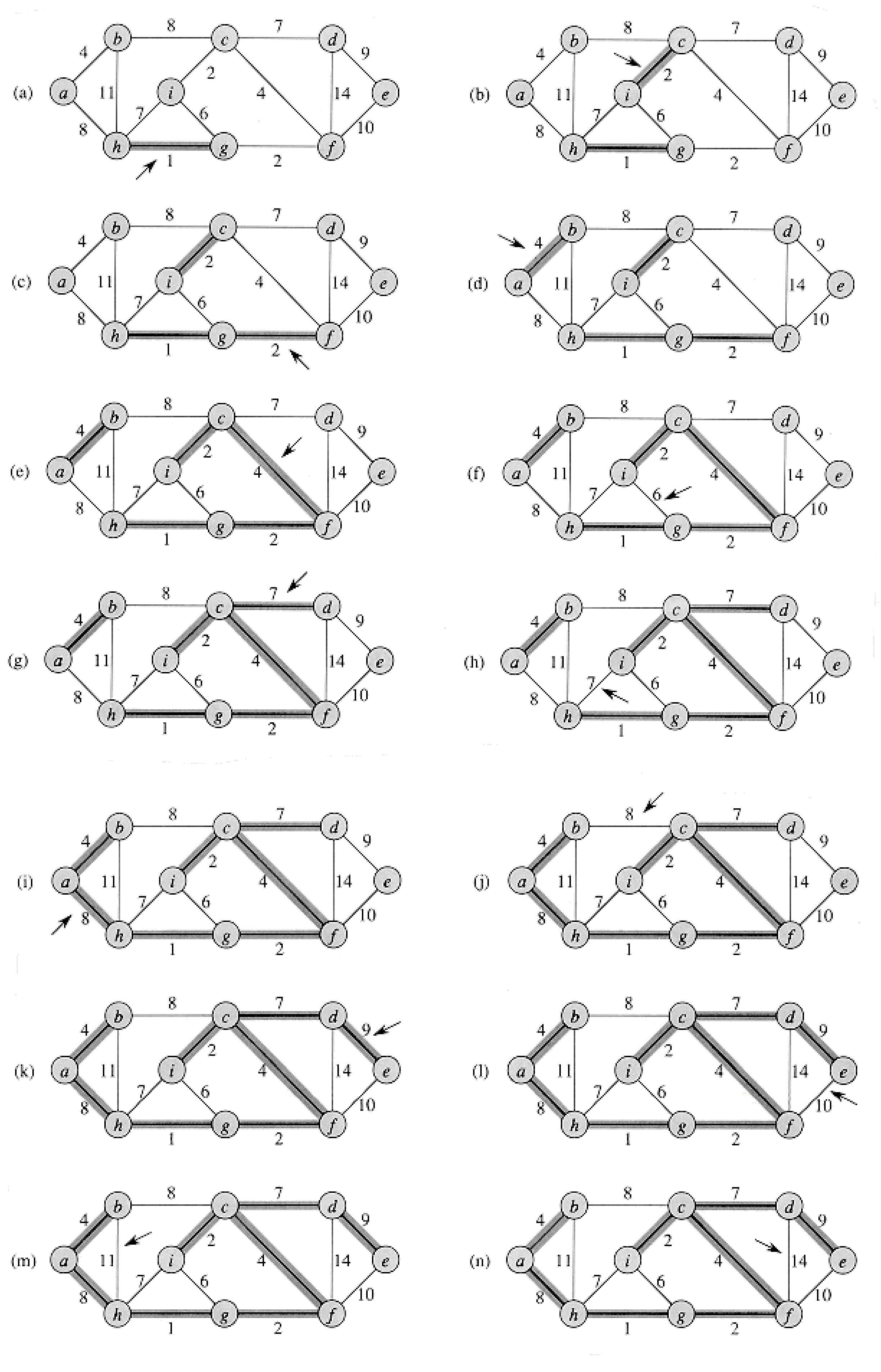


Figure 4

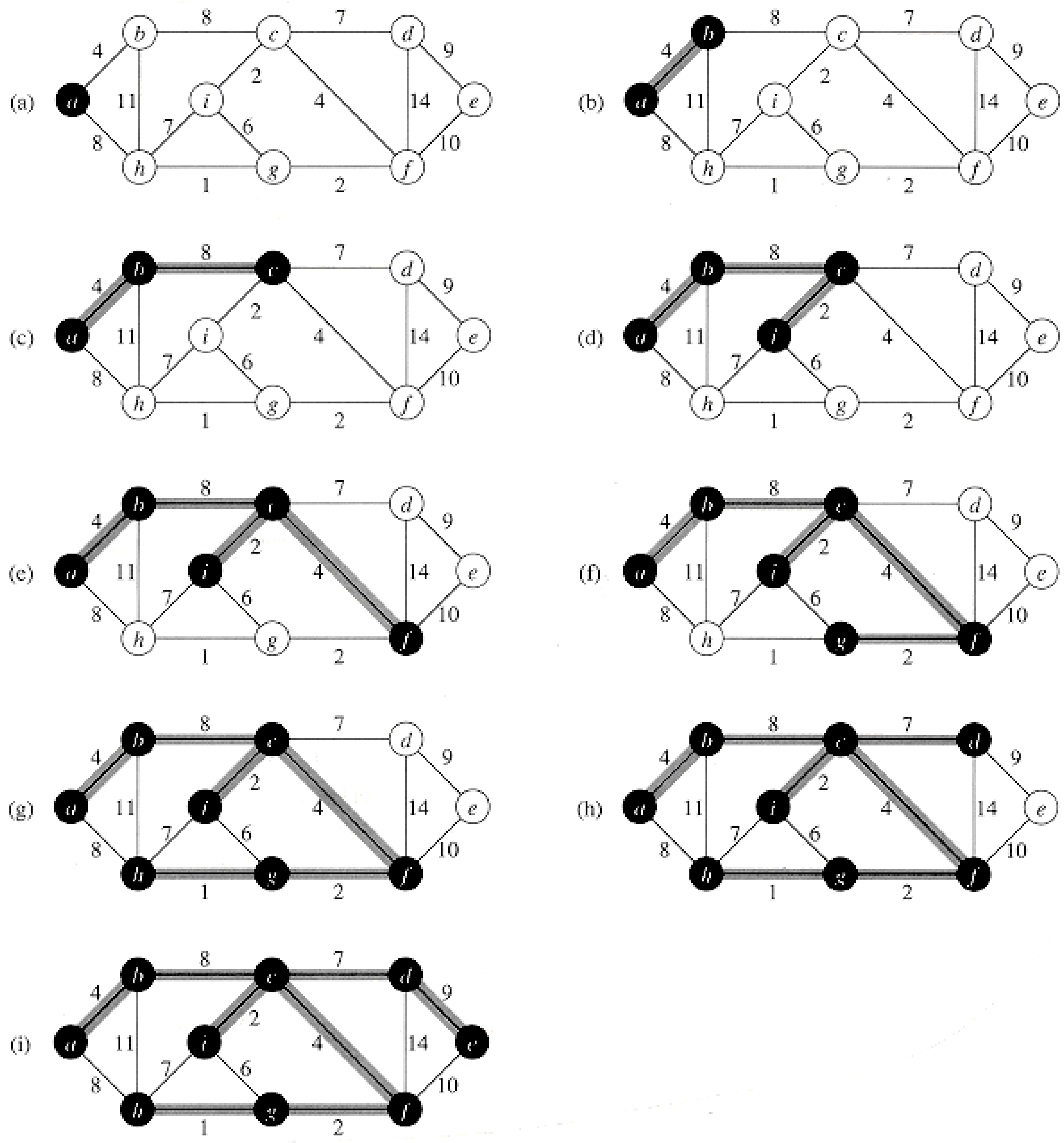


Figure 5

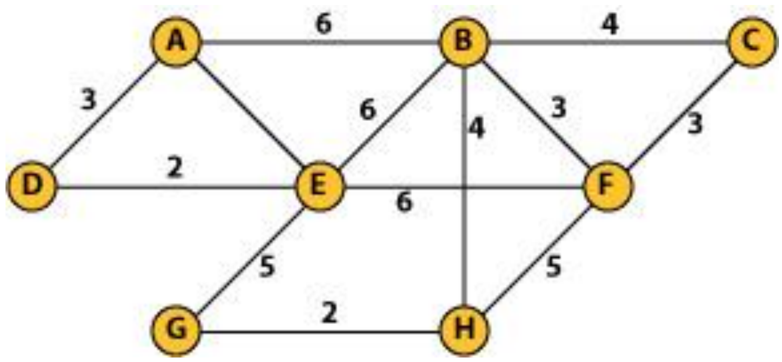


Figure 6

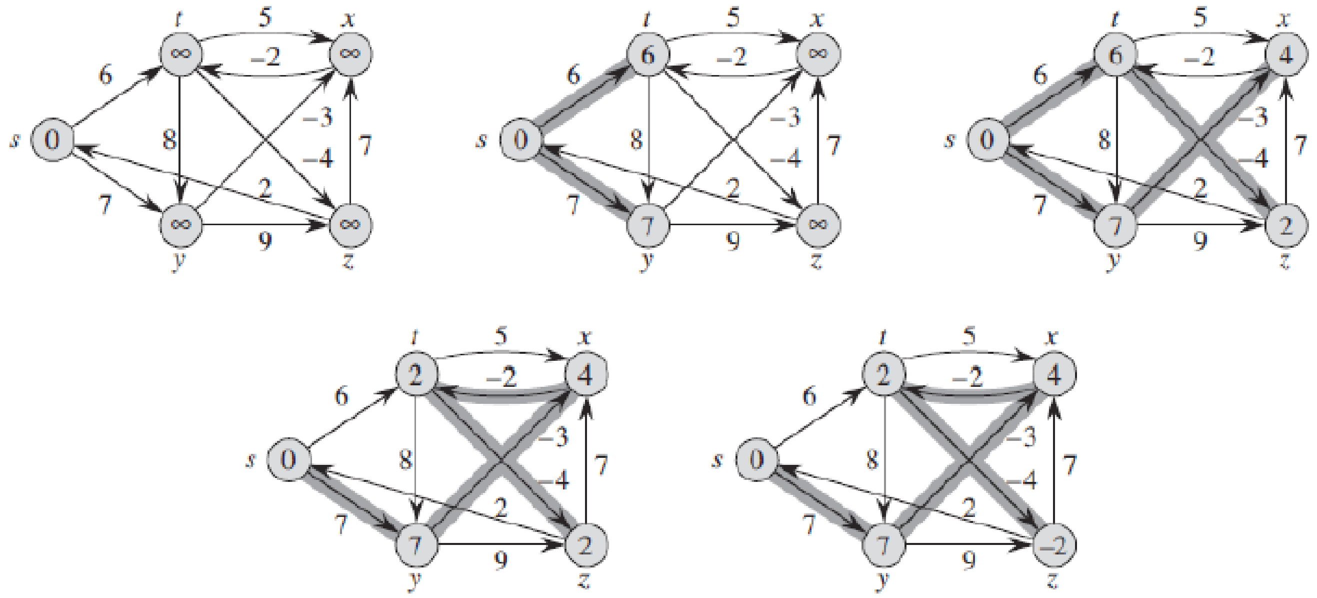


Figure 7

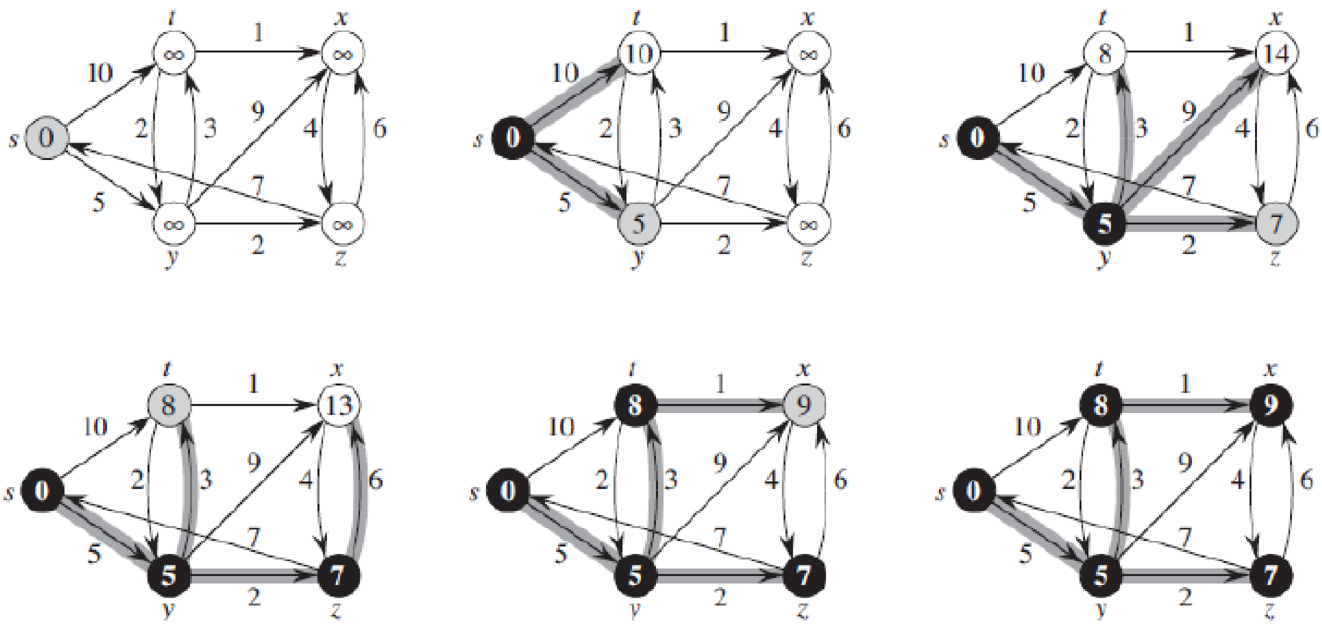


Figure 8

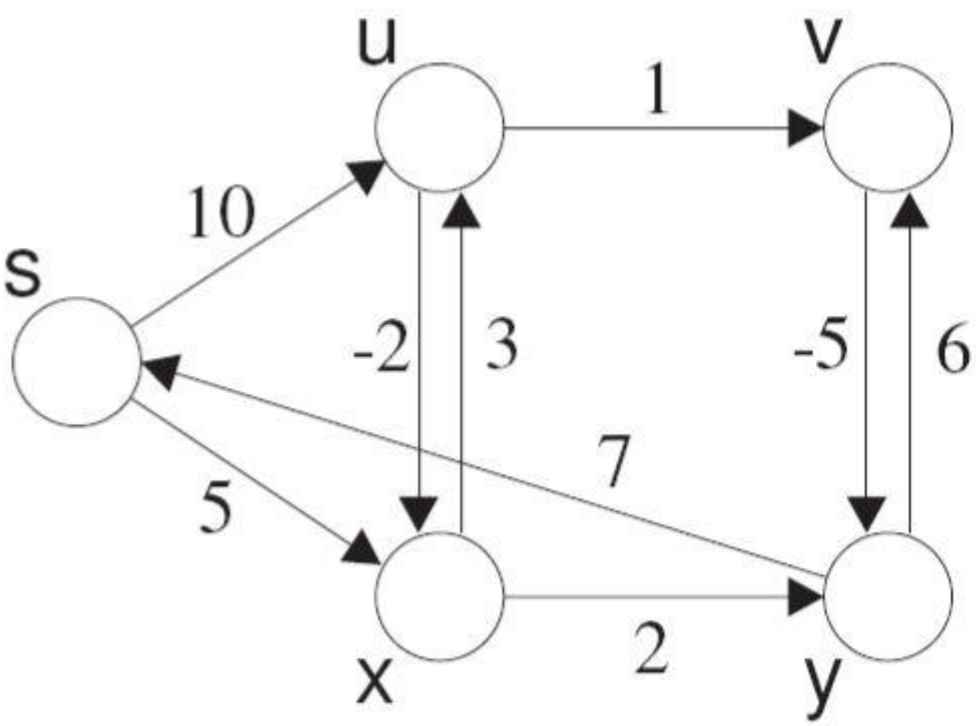


Figure 9

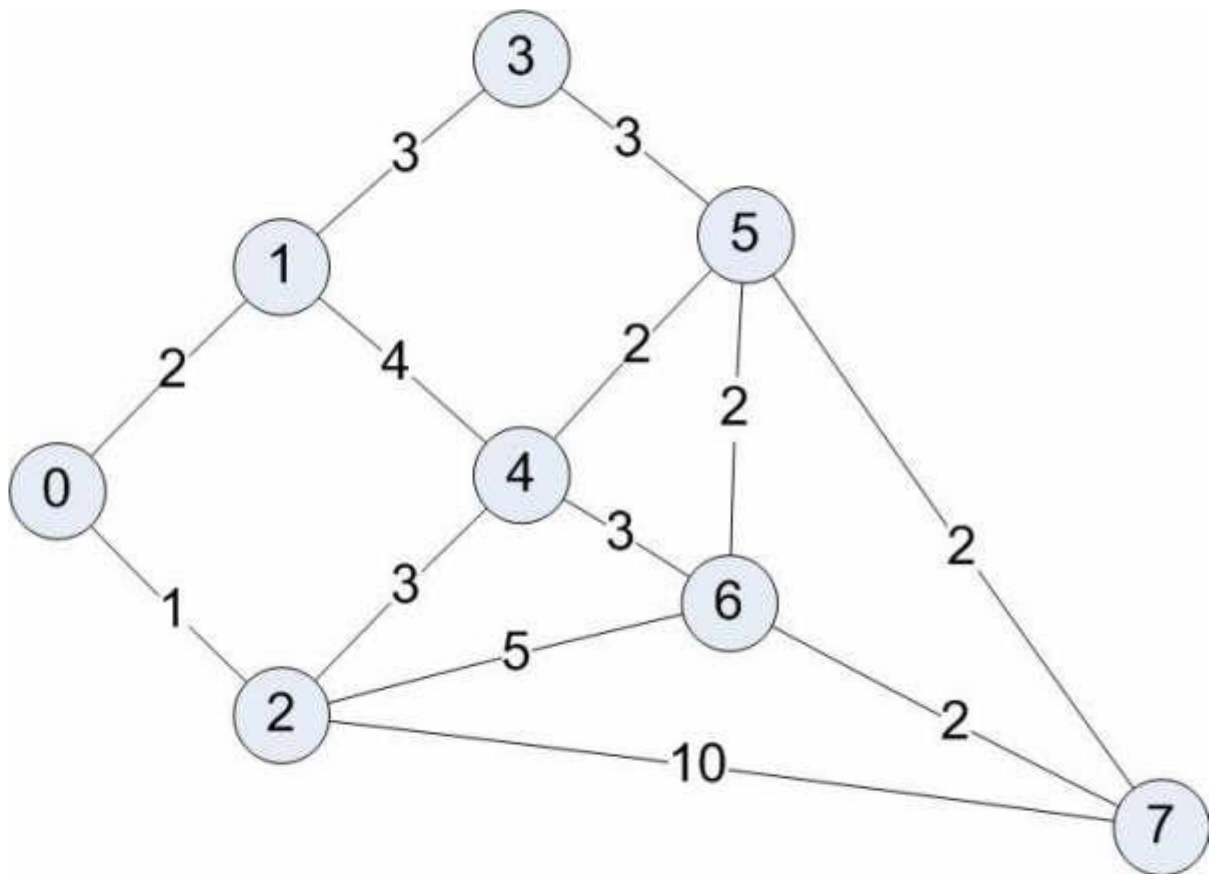
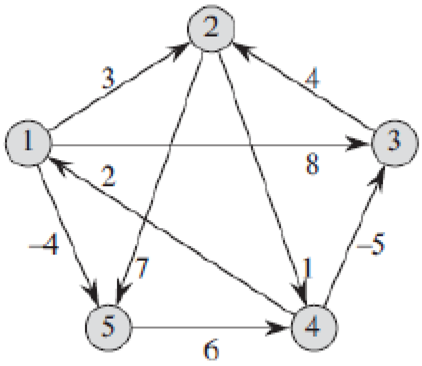


Figure 10



$$D^{(0)} = \begin{pmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{pmatrix} \qquad D^{(3)} = \begin{pmatrix} 0 & 3 & 8 & 4 & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & 5 & 11 \\ 2 & -1 & -5 & 0 & -2 \\ \infty & \infty & \infty & 6 & 0 \end{pmatrix}$$

$$D^{(1)} = \begin{pmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & 5 & -5 & 0 & -2 \\ \infty & \infty & \infty & 6 & 0 \end{pmatrix} \qquad D^{(4)} = \begin{pmatrix} 0 & 3 & -1 & 4 & -4 \\ 3 & 0 & -4 & 1 & -1 \\ 7 & 4 & 0 & 5 & 3 \\ 2 & -1 & -5 & 0 & -2 \\ 8 & 5 & 1 & 6 & 0 \end{pmatrix}$$

$$D^{(2)} = \begin{pmatrix} 0 & 3 & 8 & 4 & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & 5 & 11 \\ 2 & 5 & -5 & 0 & -2 \\ \infty & \infty & \infty & 6 & 0 \end{pmatrix} \qquad D^{(5)} = \begin{pmatrix} 0 & 1 & -3 & 2 & -4 \\ 3 & 0 & -4 & 1 & -1 \\ 7 & 4 & 0 & 5 & 3 \\ 2 & -1 & -5 & 0 & -2 \\ 8 & 5 & 1 & 6 & 0 \end{pmatrix}$$

Figure 11