CENG 511: Algorithms Analysis and Design **22.05.2014**

Problems

1. Using Figure 1 as a model, illustrate the operation of Counting-Sort on the array A=(6, 0, 2, 0, 1, 3, 4, 6, 1, 3, 2).

2. Describe an algorithm that, given n integers in the range 0 to k, preprocesses its input and then answers any query about how many of the n integers fall into a range [a ... b] in O(1) time. Your algorithm should use (n + k) preprocessing time.

3. Using Figure 2 as a model, illustrate the operation of RADIX-SORT on the following list of English words: COW, DOG, SEA, RUG, ROW, MOB, BOX, TAB, BAR, EAR, TAR, DIG, BIG, TEA, NOW, FOX.

4. Suppose that we have an array of n data records to sort and that the key of each record has the value 0 or 1. An algorithm for sorting such a set of records might possess some subset of the following three desirable characteristics:

> **1.** The algorithm runs in O(n) time.
> **2.** The algorithm is stable.
> **3.** The algorithm sorts in place, using no more than a constant amount of storage space in addition to the original array.

a. Give an algorithm that satisfies criteria 1 and 2 above.
b. Give an algorithm that satisfies criteria 1 and 3 above.
c. Give an algorithm that satisfies criteria 2 and 3 above.

5. Suppose that you are given n red and n blue water jugs, all of different shapes and sizes. All red jugs hold different amounts of water, as do the blue ones. Moreover, for every red jug, there is a blue jug that holds the same amount of water, and vice versa. It is your task to find a grouping of the jugs into pairs of red and blue jugs that hold the same amount of water. To do so, you may perform the following operation: pick a pair of jugs in which one is red and one is blue, fill the red jug with water, and then pour the water into the blue jug. This operation will tell you whether the red or the blue jug can hold more water, or if they are of the same volume. Assume that such a comparison takes one time unit. Your goal is to find an algorithm that makes a minimum number of comparisons to determine the grouping. Remember that you may not directly compare two red jugs or two blue jugs. Describe a deterministic algorithm that uses $O(n^2)$ comparisons to group the jugs into pairs.

6. Is the sequence < 23, 17, 14, 6, 13, 10, 1, 5, 7, 12 > a max-heap? (Use Figure 3)

7. Figure 4 illustrates the action of MAX-HEAPIFY(A, 2). Using Figure 4 as a model, illustrate the operation of MAX-HEAPIFY(A, 3) on the array
A = < 27, 17, 3, 16, 13, 10, 1, 5, 7, 12, 4, 8, 9, 0 >.

8. Using Figure 5 as a model, illustrate the operation of BUILD-MAX-HEAP on the array A = < 5, 3, 17, 10, 84, 19, 6, 22, 9 >.

9. Figure 6 illustrates the action of FIB-HEAP-EXTRACT-MIN. Using Figure 6 as a model, illustrate the operation of FIB-HEAP-EXTRACT-MIN on the Fibonacci heap shown in Figure 7.

10. Figure 6 illustrates the action of FIB-HEAP-EXTRACT-MIN. Using Figure 6 as a model, illustrate the operation of FIB-HEAP-EXTRACT-MIN on the Fibonacci heap shown in Figure 8.

Figures

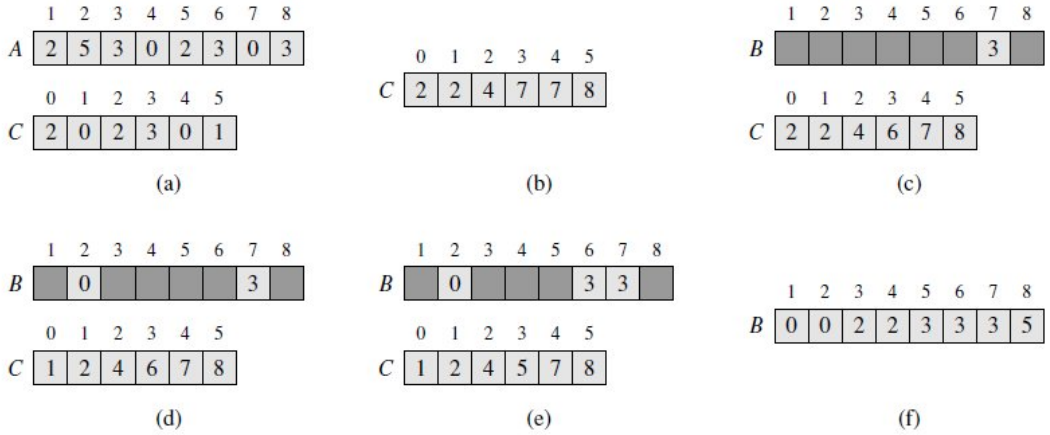

Figure 1

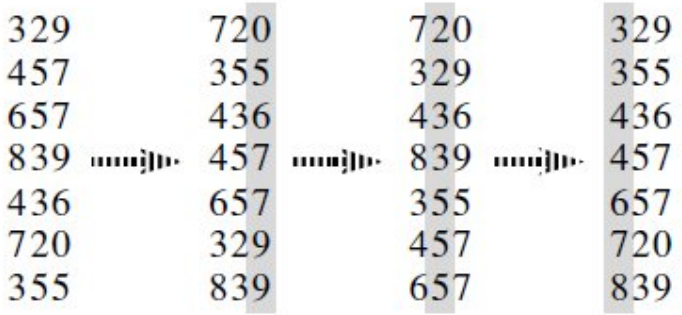

Figure 2

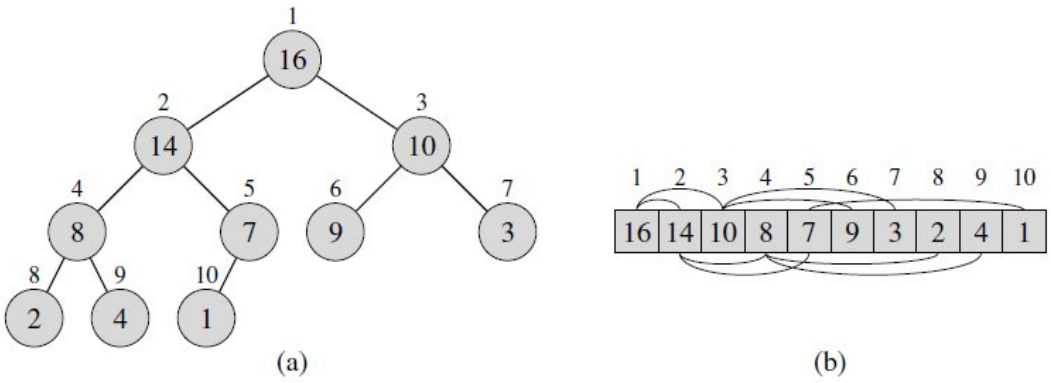

Figure 3

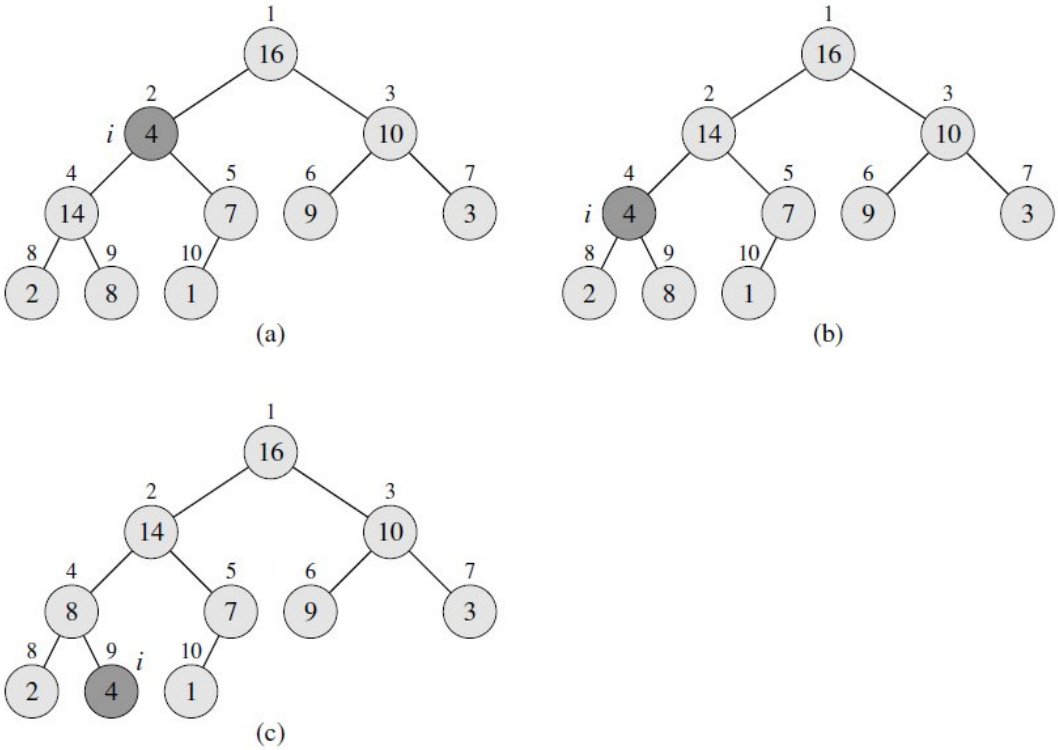

Figure 4

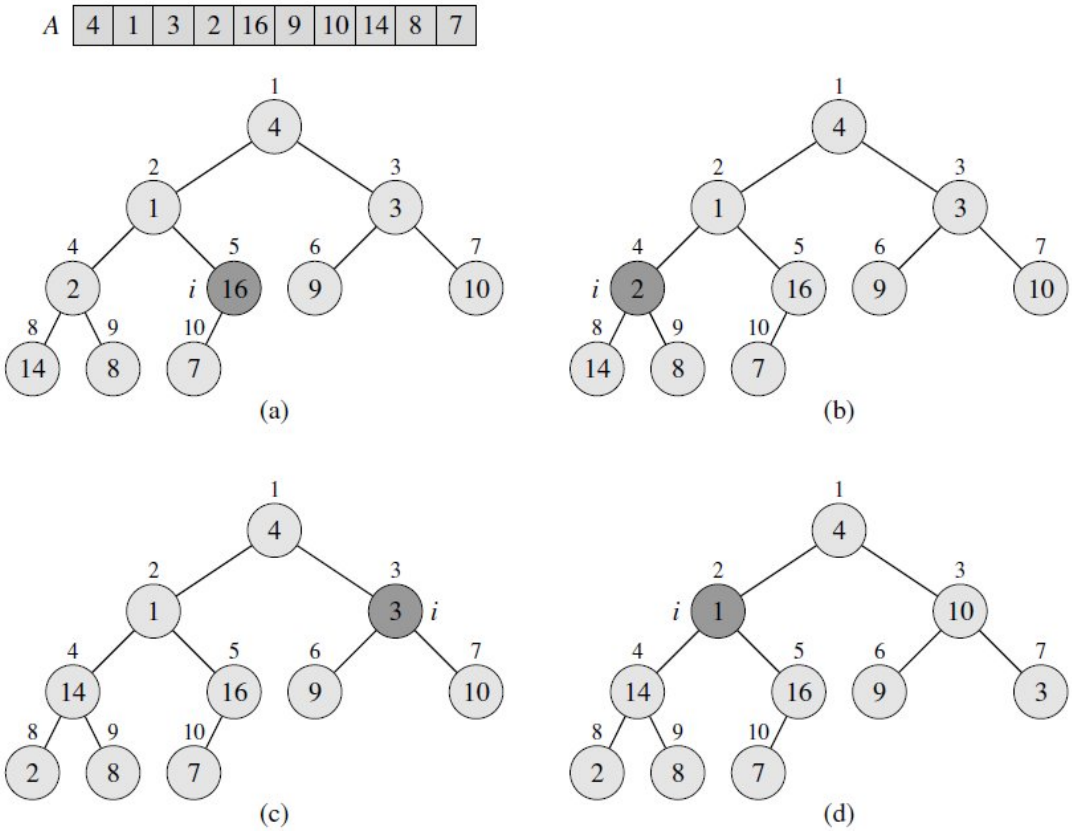

Figure 5

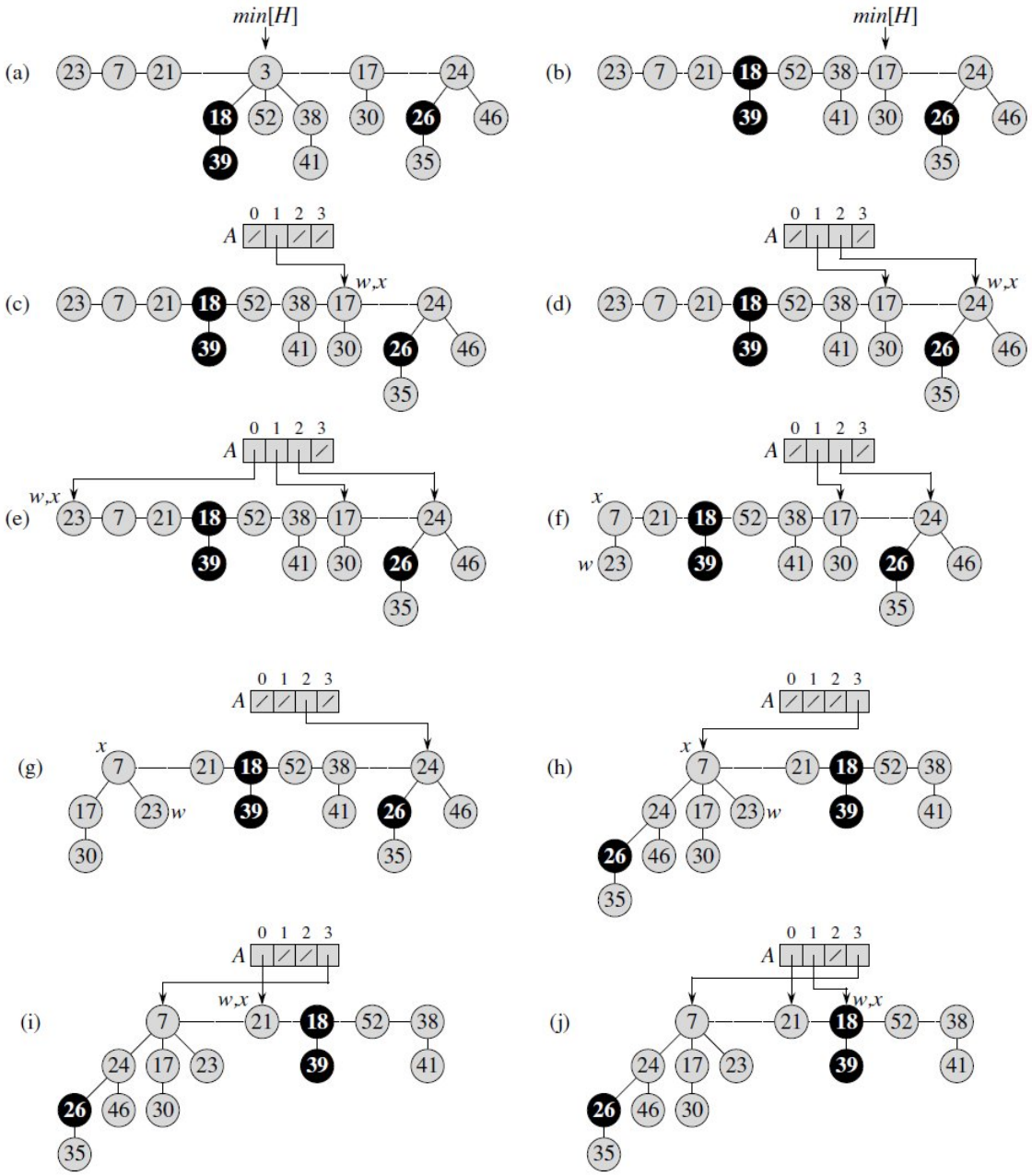

Figure 6 (part 1)

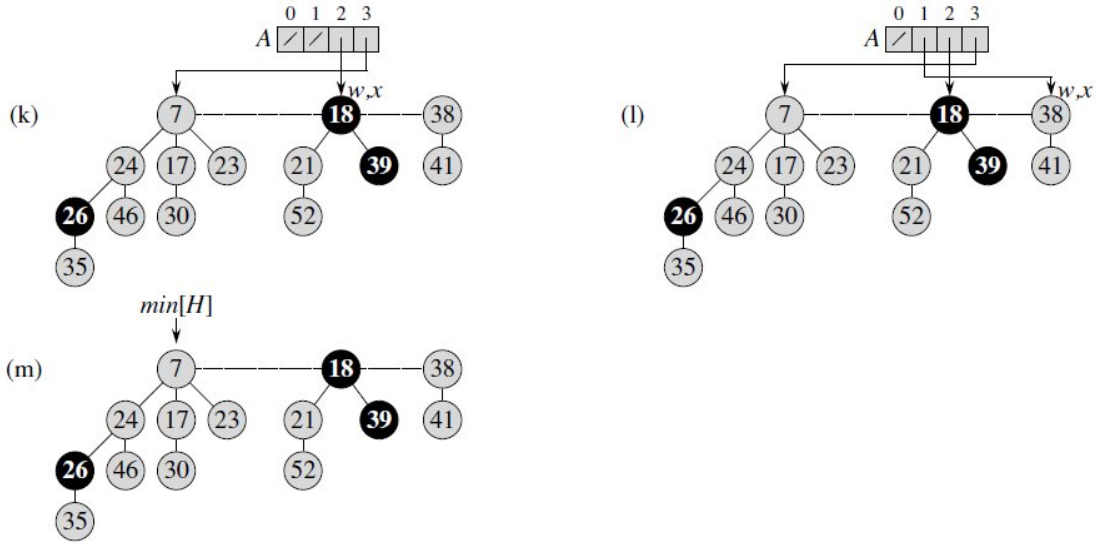

Figure 6 (part 2)

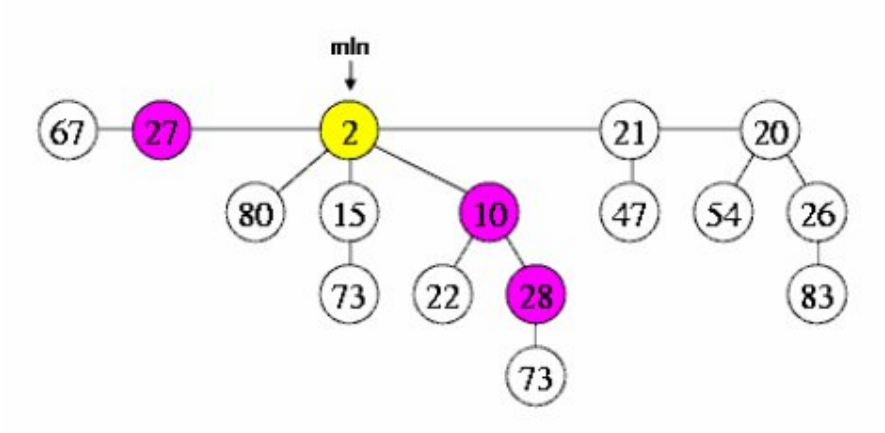

Figure 7

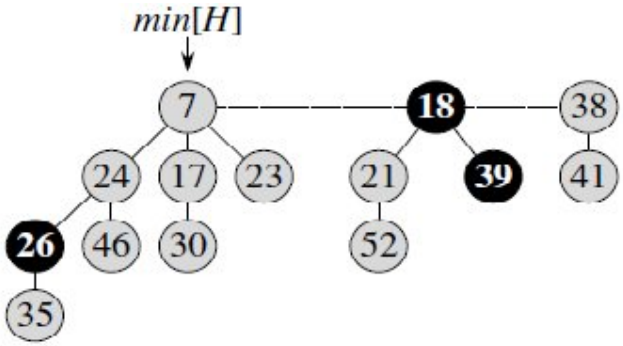

Figure 8